EXERCISE – III | SUBJECTIVE QUESTIONS

[FORMATION AND VARIABLES SEPARABLE]

1. State the order & degree of the following differential equations :

(i) $\left[\dfrac{d^2x}{dt^2}\right]^3 + \left[\dfrac{dx}{dt}\right]^4 - xt = 0$

(ii) $\dfrac{d^2y}{dx^2} = \left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}$

2. $\dfrac{\ell n(\sec x + \tan x)}{\cos x}dx = \dfrac{\ell n(\sec y + \tan y)}{\cos y}dy$

3. $\dfrac{dy}{dx} + \dfrac{\sqrt{(x^2-1)(y^2-1)}}{xy} = 0$

4. $\dfrac{dy}{dx}$ = sin (x + y) + cos (x + y)

5. (a) $\dfrac{dy}{dx} + \sin\dfrac{x+y}{2} = \sin\dfrac{x-y}{2}$

(b) sin x . $\dfrac{dy}{dx}$ = y. ℓny if y = e, when x = $\dfrac{\pi}{2}$

6. $e^{(dy/dx)}$ = x + 1 given that when x = 0, y = 3

7. The population P of a town decreases at a rate proportional to the number by which the population exceeds 1000, proportionality constant being k > 0. Find

(a) Population at any time t, given initial population of the town being 2500.

(b) If 10 years later the population has fallen to 1900, find the time when the population will be 1500.

(c) Predict about the population of the town in the long run.

8. It is known that the decay rate of radium is directly proportional to its quantity at each given instant. Find the law of variation of a mass of radium as a function of time if at t = 0, the mass of the radius was m_0 and during time t_0 α% of the original mass of radium decay.

9. A normal is drawn at a point P(x, y) of a curve. It meets the x-axis at Q. If PQ is of constant length k, then show that the differential equation describing such curves is, $y\dfrac{dy}{dx} = \pm\sqrt{k^2 - y^2}$. Find the equation of such a curve passing through (0, k).

10. Find the curve for which the sum of the lengths of the tangent and subtangent at any of its point is proportional to the product of the co-ordinates of the point of tangency, the proportionality factor is equal to k.

11. Find the curve y = f(x) where f(x) ≥ 0, f(0) = 0, bounding a curvilinear trapezoid with the base [0, x] whose area is proportional to $(n + 1)^{th}$ power of f (x). It is known that f(1) = 1.

12. A curve is such that the length of the polar radius of any point on the curve is equal to the length of the tangent drawn at this point. Form the differential equation and solve it to find the equation of the curve.

[HOMOGENEOUS]

13. (a) $\dfrac{dy}{dx} = \dfrac{x^2 + xy}{x^2 + y^2}$

(b) $(x^3 - 3xy^2)\,dx = (y^3 - 3x^2y)\,dy$

14. Find the equation of a curve such that the projection of its ordinate upon the normal is equal to its abscissa.

15. The light rays emanating from a point source situated at origin when reflected from the mirror of a search light are reflected as beam parallel to the x-axis. Show that the surface is parabolic, by first forming the differential equation and then solving it.

16. The perpendicular from the origin to the tangent at any point on a curve is equal to the abscissa of the point of contact. Find the equation of the curve satisfying the above condition and which passes through (1, 1).

17. Show that the equation of the curve intersecting with the x-axis at the point $x = 1$ and for which the length of the subnormal at any point of the curve is equal to the arithmetic mean of the co-ordinates of this point is $(y - x)^2 (x + 2y) = 1$.

18. Use the substitution $y^2 = a - x$ to reduce the equation $y^3 \cdot \frac{dy}{dx} + x + y^2 = 0$ to homogeneous form and hence solve it.

19. $\left[x\cos\frac{y}{x} + y\sin\frac{y}{x}\right] y = \left[y\sin\frac{y}{x} - x\cos\frac{y}{x}\right] x\frac{dy}{dx}$

20. Find the curve for which any tangent intersects the y-axis at the point equidistant from the point of tangency and the origin.

21. $(x - y)\, dy = (x + y + 1)\, dx$

22. $\frac{dy}{dx} = \frac{x + y + 1}{2x + 2y + 3}$

23. Show that the curve such that the distance between the origin and the tangent at an arbitrary point is equal to the distance between the origin and the normal at the same point.

$$\sqrt{x^2 + y^2} = ce^{\pm\tan^{-1}\frac{y}{x}}$$

[LINEAR]

24. Let $y = y(t)$ be a solution to the differential equation $y' + 2t\,y = t^2$, then find $\lim\limits_{t\to\infty}\frac{y}{t}$.

25. $(1 - x^2)\frac{dy}{dx} + 2xy = x(1 - x^2)^{1/2}$

26. Find the curve such that the area of the trapezium formed by the co-ordinate axes, ordinate of an arbitrary point and the tangent at this point equals half the square of its abscissa.

27. Find the curve possessing the property that the intercept, the tangent at any point of a curve cuts off on the y-axis is equal to the square of the abscissa of the point of tangency.

28. $x\frac{dy}{dx} - y = 2x^2 \operatorname{cosec} 2x$

29. $(1 + y^2)\, dx = (\tan^{-1} y - x)dy$

30. Find the curve such that the area of the rectangle constructed on the abscissa of any point and the initial ordinate of the tangent at this point is equal to a^2. (Initial ordinate means y intercept of the tangent).

31. Let the function $\ln f(x)$ is defined where $f(x)$ exists for $x \geq 2$ and k is fixed positive real number, prove that if $\frac{d}{dx}(x.\, f(x)) \leq -k\, f(x)$ then $f(x) \leq A\, x^{-1-k}$ where A is independent of x.

32. Find the differentiable function which satisfies the equation $f(x) = -\int_0^x f(t)\tan t\,dt + \int_0^x \tan(t - x)dt$ where $x \in (-\pi/2, \pi/2)$

33. Find all functions $f(x)$ defined on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ with real values and has primitive $F(x)$ such that

$f(x) + \cos x \cdot F(x) = \frac{\sin 2x}{(1+\sin x)^2}$. Find $f(x)$.

34. A tank contains 100 litres of fresh water. A solution containing 1 gm/litre of soluble lawn fertilizer runs into the tank at the rate of 1 lit/min and the mixture is pumped out of the tank of 3 litres/min. Find the time when the amount of fertilizer in the tank is maximum.

35. A tank with a capacity of 1000 litres originally contains 100 gms of salt dissolved in 400 litres of water. Beginning at time $t = 0$ and ending at time $t = 100$ minutes, water containing 1 gm of salt per litre enters the tank at the rate of 4 litre/minute and the wheel mixed solution is drained from the tank at a rate of 2 litre/minute. Find the differential equation for the amount of salt y in the tank at time t.

[GENERAL - CHANGE OF VARIABLE BY A SUITABLE SUBSTITUTION]

36. $(x - y^2)\, dx + 2xy\, dy = 0$

37. $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1 + x)\, e^{x} \sec y$

38. $\frac{dy}{dx} = \frac{e^{y}}{x^{2}} - \frac{1}{x}$

39. $\left(\frac{dy}{dx}\right)^{2} - (x+y)\frac{dy}{dx} + xy = 0$

40. $\frac{dy}{dx} = \frac{y^{2} - x}{2y(x+1)}$

41. $\frac{dy}{dx} = e^{x-y}(e^{x} - e^{y})$

42. $yy' \sin x = \cos x\, (\sin x - y^{2})$